

United Kingdom
Mathematics Trust

INTERMEDIATE MATHEMATICAL OLYMPIAD

HAMILTON PAPER

Thursday 16 March 2023

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England & Wales: Year 10
Scotland: S3
Northern Ireland: Year 11

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. Start each question on an official answer sheet on which there is a QR code
5. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ‘:’ from the question’s answer sheet QR code. Please do not write your name or initials on additional sheets. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π , fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

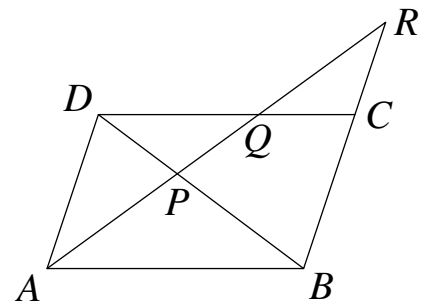
challenges@ukmt.org.uk

www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as π , fractions, or square roots, if appropriate, but not decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do not hand in rough work.*

1. Susie thinks of a positive integer n . She notices that, when she divides 2023 by n , she is left with a remainder of 43. Find how many possible values of n there are.
2. The two positive integers a, b with $a > b$ are such that $a\%$ of $b\%$ of a and $b\%$ of $a\%$ of b differ by 0.003. Find all possible pairs (a, b) .
3. The n th term of a sequence is the first non-zero digit of the decimal expansion of $\frac{1}{\sqrt{n}}$.
How many of the first one million terms of the sequence are equal to 1?

4. In the parallelogram $ABCD$, a line through A meets BD at P , CD at Q and BC extended at R . Prove that $\frac{PQ}{PR} = \left(\frac{PD}{PB}\right)^2$.



5. Mickey writes down on a board n consecutive whole numbers the smallest of which is 2023. He then replaces the largest two numbers on the board with their difference, reducing the number of numbers on the board by one. He does this repeatedly until there is only a single number on the board. For which values of n is this last remaining number 0?
6. Find all triples (m, n, p) which satisfy the equation

$$p^n + 3600 = m^2$$

where p is prime and m, n are positive integers.